Solution to Exercise 8.8 (Version 1, 10/7/15)

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Exercise 8.8*

Consider the data from the calcium pot trial of Example 4.1 (Table 4.1). In this trial, the treatments A, B, C and D were concentrations of calcium in the soil, measured as relative concentrations of 1, 5, 10 and 20, respectively. Re-analyse these data using polynomial contrasts. Which low-order polynomial provides the best fit to these data?

Solution 8.8

The model for this experiment was written in Example 4.1 as

Response variate: *Length*
Structural component: Pot
Explanatory component: [1] + Calcium

The calcium concentrations were originally coded as A, B C and D. We need to add information on the numerical values of concentrations (A=1, B=5, C=10, D=20) into the analysis to fit polynomial contrasts; the way this is done varies between software (see program files for details). There are four different concentrations, giving 3 df, so we can define 3 polynomial contrasts: linear, quadratic and cubic. Coefficients of the orthogonal polynomials for these concentrations (with equal replication) are shown in Table S8.8.1. Before deciding what order of polynomial contrast to fit, we consider a plot of the data, as shown in Figure S8.8.1. It is clear that the trend is non-linear, so fitting solely linear trend is unrealistic. Conversely, if we fit a cubic polynomial then it will fit exactly and there is no objective way to evaluate its fit. We therefore fit a quadratic polynomial and use the cubic trend as a measure of lack of fit. The ANOVA table partitioned into these contrasts is shown in Table S8.8.2.

Table S8.8.1. Coefficients of orthogonal polynomial contrasts for calcium concentrations.

Calcium	Coefficient of orthogonal polynomial		
concentration	Linear	Quadratic	Cubic
1	-0.2517	0.2516	-0.1528
5	-0.1259	-0.1131	0.3484
10	0.0315	-0.3083	-0.2323
20	0.3461	0.1670	0.0367

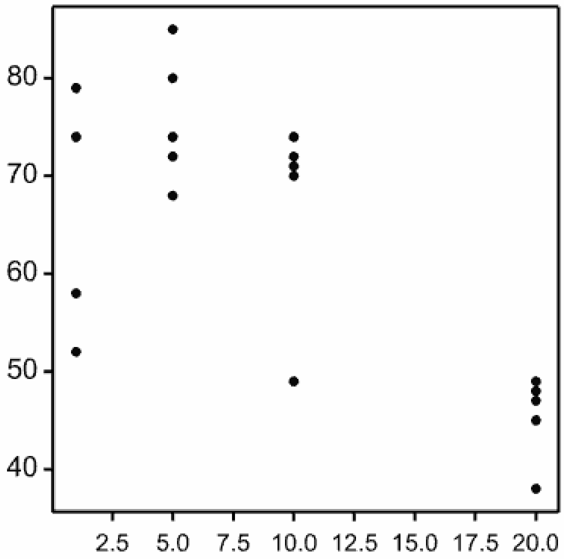

Figure S8.8.1. Root lengths from calcium pot trial plotted against calcium concentration.

Table S8.8.2. ANOVA table for root lengths using polynomial contrasts.

Source of variation	df	Sum of squares	Mean square	Variance ratio	P
Calcium	3	2462.950	820.983	10.753	>0.001
Linear	1	1549.506	1549.506	$F^{lin} = 20.295$	>0.001
Quadratic	1	737.218	737.218	$F^{quad} = 9.656$	0.007
Deviations	1	176.227	176.227	$F^{dev} = 2.308$	0.148
Residual	16	1221.600	76.350		
Total	19	3684.550			

There is strong evidence that both the linear and quadratic polynomial components are required to describe the pattern (linear contrast, $F^{lin} = 20.295$ with 1 and 16 df, $P > 0.001$; quadratic contrast, $F^{quad} = 9.656$ with 1 and 16 df, $P = 0.007$). The remaining contrast is not statistically significant ($F^{dev} = 2.308$ with 1 and 16 df, $P = 0.148$), indicating no evidence of lack of fit to a pattern of quadratic trend.

The fitted quadratic trend can be written (in terms of powers of the concentrations rather than as coefficients of the orthogonal polynomials) as

$$Length(c) = 64.21 + 2.18c - 0.16c^2$$

where *Length* is the predicted root length as a function of calcium concentration *c*. This function is only valid over the range of concentrations tested, ie. from 1 to 20 units. This fitted quadratic model is shown with the data and treatment means in Figure S8.8.2, and gives a reasonable approximation to the pattern of treatment means. Given the small number of concentrations tested, we should be cautious about believing this as a true model for the response. In particular, we might be sceptical about interpolation between concentrations of 10 and 20, where there are no observations to support (or contradict) the model. However, the quadratic model does give a reasonable summary of the data observed.

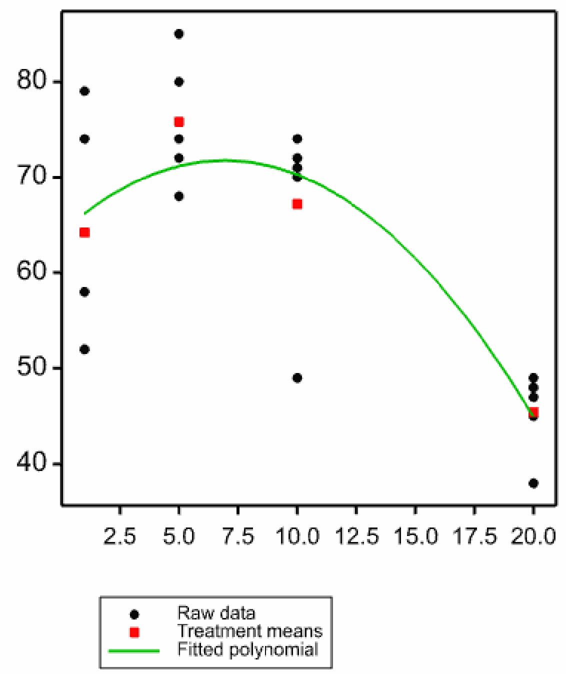

Figure S8.8.2. Root lengths (black dots) plotted against calcium concentration with fitted means (red squares) and fitted quadratic polynomial model (green line).