Solution to Exercise 11.6 (Version 1, 1/8/15)

from Statistical Methods in Biology: Design & Analysis of Experiments and Regression (2014) S.J. Welham, S.A. Gezan, S.J. Clark & A. Mead. Chapman & Hall/CRC Press, Boca Raton, Florida. ISBN: 978-1-4398-0878-8

© S J Welham, S A Gezan, S J Clark & A Mead, 2015.

Exercise 11.6

Five pruning treatments were tested on apple trees (Pearce, 1965, Section 6.2). A balanced incomplete block design was used to allocate the five treatments (a–e) to four branches on each of 15 trees (60 branches in total). One of the outcomes measured was the length of shoots from the middle third of each branch, but this was only measured for treatments a, b and d. The shoot lengths (*Length*) are in file SHOOT.DAT with the unit numbers (*ID*), structural factors (Tree, Branch) and treatment factor (Treatment). Analyse these data, accounting for possible differences between trees as well as treatments. Can you identify which treatment produces the longest shoots? (We re-visit these data in Exercise 16.3.)

Data 11.6 (SHOOT.DAT, data from S.C. Pearce, *Biological Statistics: An Introduction*, 1965, Sec. 6.2)

ID	Tree	DBranch	Treatment	Length	ID	Tree	DBranch	Treatment	Length
1	1	1	a	87	19	8	2	b	85
2	1	2	b	74	20	8	3	d	73
3	1	3	d	80	21	9	1	b	80
4	2	1	a	87	22	9	2	d	64
5	2	2	d	75	23	10	1	a	83
6	3	1	a	83	24	10	2	d	90
7	3	2	b	77	25	11	1	b	82
8	3	3	d	54	26	11	2	d	93
9	4	1	a	88	27	12	1	a	92
10	4	2	b	85	28	12	2	b	87
11	5	1	a	91	29	12	3	d	66
12	5	2	d	86	30	13	1	a	82
13	6	1	a	79	31	13	2	b	76
14	6	2	b	78	32	13	3	d	87
15	6	3	d	64	33	14	1	b	76
16	7	1	a	87	34	14	2	d	89
17	7	2	b	85	35	15	1	a	104
18	8	1	a	80	36	15	2	b	73

Solution 11.6

As explained in the question, this data set is a subset of a BIBD with 5 pruning treatments and 15 blocks of size 4, with 15 trees as blocks and branches as experimental units, with 4 branches used per tree. This means one treatment was omitted on each tree. As shoot length was only measured for 3 treatments, we have data from either 2 or 3 branches per tree and the structure is no longer balanced.

The structural component of the model has branches nested within trees, and can be written as

Structural component: Tree/DBranch

The explanatory component of the model just contains the overall mean and effects of pruning treatments, and can be written as

Explanatory component: [1] + Treatment

To estimate treatment effects after accounting for blocking (trees), ie. the intra-block analysis, we combine the structural and explanatory components and fit the blocks before the treatments, giving the model

Response variable: *Length*
Explanatory component: [1] + Tree + Treatment

The residual plots from this model are shown in Figure S11.6.1 and show no cause for concern.

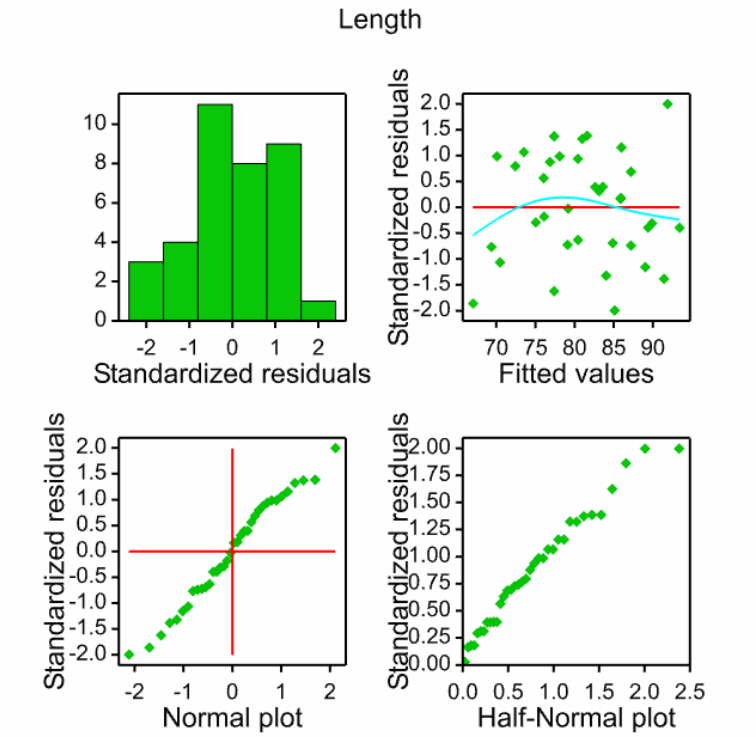

Figure S11.6.1 Composite set of residual plots (based on standardized residuals) for shoot length.

The ANOVA table from this analysis is in Table S11.6.1. After accounting for tree effects, the variance ratio for treatments ($F_{2,19}$ = 3.249, P = 0.061) gives no evidence of any difference in population mean shoot length between treatments. Table S11.6.2 shows the predicted means for the three pruning treatments, but without a significant F-test, we have no grounds for suggesting that any one treatment gives longer shoots than another. Pairwise comparisons via Tukey's simultaneous confidence intervals confirm this assertion.

Table S11.6.1 Sequential ANOVA table for intra-block model for shoot length.

Source of variation	df	Sum of squares	Mean square	Variance ratio	*P*
+Tree	14	1100.00	78.57	0.965	0.518
+Treatment	2	529.33	264.67	3.249	0.061
Residual	19	1547.67	81.46		
Total	35	3177.00			

Table S11.6.2 Predicted mean shoot lengths for pruning treatments. SED = 3.94 with 19 df.

a	b	d
87.3	80.6	77.5